THE METHOD OF EQUIVALENT FOUR-POLE NETWORKS APPLIED TO SIMULATION OF COMPLEX BULK ACOUSTIC RESONATORS.

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ABSTRACT

The paper presents the results of simulation of an acoustic FTIR laser Q-switch. Being a complex bulk acoustic resonator, it consists of active and passive piezoelectric transducers, and acoustic layers, where may occur some complex acoustic processes, like acoustic mode conversion. The output optical signal is a result of the response to an electrical signal.

The simulation has been made using the frequency-domain analysis. For each element of the Q-switch, an equivalent four-pole network has been composed, and its transmission matrix has been found for each acoustic mode traveling in the switch. Using well-known rules for matrix analysis for such networks, the frequency-dependent transmission coefficients and input impedance have been found for each equivalent network. Using those coefficient and spectrum of input signal, the spectrum of output response for the equivalent networks have been found. The output signal is found as a sum of the responses of two equivalent networks.

As the result, time plots of output signal for the device and its input impedance have been obtained. These results are in good agreement with experimental measurements. All this proves that the method of equivalent networks can be used for the simulation of complex bulk resonators.

KEYWORDS: acoustics, FTIR Q-switch, equivalent network, simulation, bulk resonator

1. FTIR Q-SWITCH

The Q-switch being simulated is shown in Fig. 1.

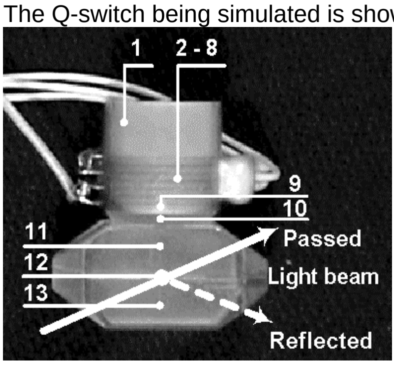


Fig. 1
An acoustic FTIR laser Q-switch.

1. Counterweight (Passive piezoceramics)
2. – 8. Six piezoelectric transducer modules (See Fig. 4)
9. Passive piezoceramic layer
10. Top glass plate
11. Glass prism
12. Air gap
13. Bottom glass prism.

Normally, air gap 12 between the top and bottom glass prisms is open, and the light beam reflects from the air-glass boundary at gap faces.

When an electric signal (the first pulse in Fig. 2) comes to piezoelectric transducer modules (2 – 8 in Fig. 1), the transducers generate an acoustic pulse, which travels to the air gap and closes it.

When the air gap is closed, the light beam passes through it, forming an output optical pulse (the second pulse in Fig. 2).

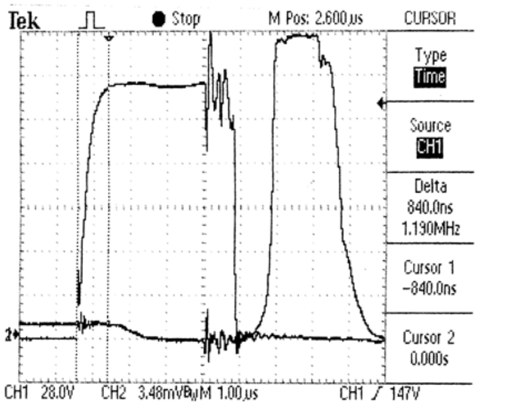


Fig. 2
Input electric signal (the first pulse) and output optical signal (the second pulse).

While traveling in the Q-switch, the acoustic pulse reflects from boundaries between different acoustic layers and faces, and a full output optical signal appears as a long series of pulses shown in Fig. 3.

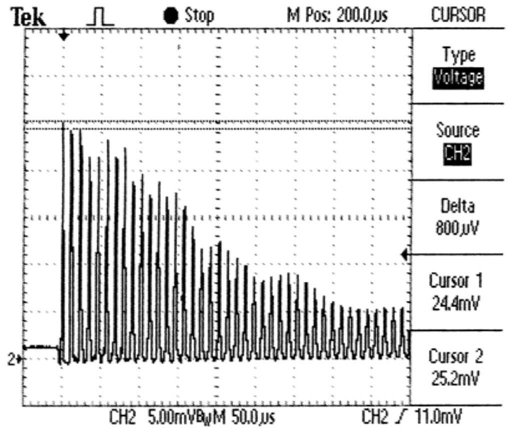


Fig. 3
Full output optical signal

More accurately, the gap thickness and optical transmission are related by the following expression:

$$T(t) = \frac{1}{\alpha \sinh\left(y\left(d_o(t)\right)\right)^2 + 1}, \qquad (1)$$

where $T(t)$ is the optical transmission coefficient, $d_0(t)$ is the instant gap thickness,

$$\alpha = \frac{\left(n^2 - 1\right)^2}{4n^2 \cos(\psi)^2 \cdot \left(n^2 \sin(\psi)^2 - 1\right)} \cdot \left[\left(n^2 + 1\right)\sin(\psi)^2 - 1\right]^2,$$

where n is the refractive index of the glass, ψ=67.38°,

$$y\left(d_o(t)\right) = 2\pi \frac{d_o(t)}{\lambda} \sqrt{n^2 \sin(\psi)^2 - 1},$$

and λ is the optical wavelength.

In Eq. 1 only one parameter $d_0(t)$ needs to be determined. It may be found as

$$d_o(t) = d_o \cdot (1 + s_o(t)), \tag{2}$$

where d_0 is the initial gap thickness and $s_0(t)$ is the gap strain caused by the acoustic pulse, therefore the problem can be reduced to the simulation of complex bulk electro-acoustical systems.

The aim of the simulation was to find the output optical response for a given input electrical signal. The best way to perform such simulation is to use the well-known method of equivalent four-pole network analysis [Ref. 1,2]. This method is based on a strong similarity between electrical and acoustical phenomena [Ref. 3], and treats an acoustic layer as a piece of a long acoustic lossy or lossless transmission line. Such acoustic transmission line can be mathematically described with its transmission matrix $\mathbf{A}$, similar to that of an electrical four-pole network. The same matrices can be written for the electrical components of the system. Than well-known rules of four-pole network matrix analysis can be applied to the entire system, giving the required output response.

The advantage of this simulation technique is that it gives a simple and rather straightforward approach to complex electro-acoustical systems, removing virtually all restrictions to their complexity, and each element in the system usually can be described mathematically rather simply. Also, the system overall characteristics, such as its transmission coefficient and input impedance, are also easy to find.

2. ACOUSTIC MODES TRAVELLING IN THE Q-SWITCH

The input signal applied to Q-switch inputs is wideband; therefore, the transducers generate acoustic waves in a wide frequency band. While traveling in a bounded medium, they form a very complex acoustic fields consisting of many acoustic modes [Ref. 3]. Mode analysis of such complex acoustic system is very difficult to perform, but some assumptions are possible:

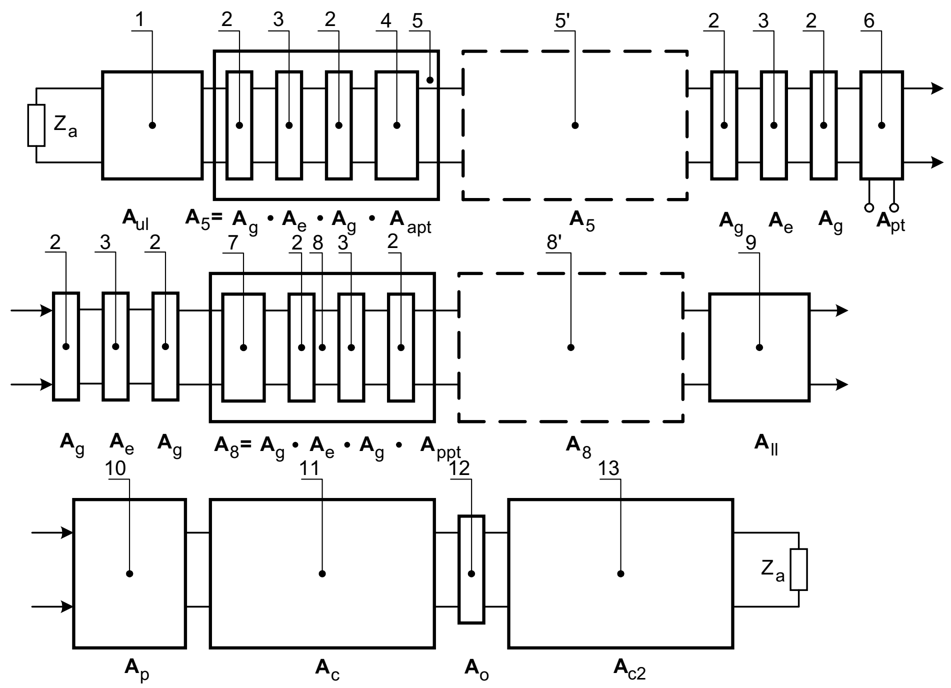
1. The system is linear, therefore, each mode travels in the system independently.
2. There are two mode types in the system: first, a conventional longitudinal mode traveling in the whole system. Second, a slender rod mode in the transducer module corresponding to the SH mode in the glass prisms.
3. Slender mode generation is a non-reciprocal process. This may be simulated by using high output impedance of signal source.
4. Once generated, these modes do not interact and do not convert into each other.

Following all these assumption, the equivalent four-pole network analysis should be applied to each mode independently, and the overall response of the system is a sum of responses from these two modes.

3. Q-SWITCH BLOCK DIAGRAM

Equivalent four-pole network analysis starts with the block diagram of the system being simulated. For the Q-switch, the system has 6 electrical inputs and one acoustic output: the mechanical strain in the center of air gap 12. As the system is assumed linear, it can be represented as six partial linear systems with one input and one output, the overall response being the sum of six partial responses of these partial systems [Ref. 4].

The equivalent network for one partial system is shown in Fig. 4.



1. Counterweight
2. Glue layer
3. Electrode
4. Active piezoelectric transducer
5. Active transducer module

5'. Active transducer modules

6. Working transducer
7. Passive transducer
8. Passive transducer module

8'. Passive transducer modules

9. Passive piezoceramics layer
10. Top glass plate
11. Glass prism
12. Air gap
13. Bottom glass prism.

Z_a is air acoustic impedance

Fig. 4
Q- switch block diagram

Each element in this diagram, be it a passive acoustic layer or electro-acoustic element, is represented as an equivalent four-pole network described by its 2x2 transmission matrix $\mathbf{A}$. The electrical input for the system is working transducer 6, which in fact, is a six-pole network, but it will be shown below how it may be converted into a four-pole network. All the networks are connected serially, thus the overall transmission matrix $\mathbf{A}_\Sigma$

is a product of all transmission matrices of correspondent elements. Also, the multiplication of matrices is an associative operation, thus matrices can be multiplied partially, forming transmission matrices for common modules, such as passive or active transducer modules.

Using the well-known electrical and mechanical analogies (Ref. 3), the transmission coefficient for the system may be written as

$$\dot{K}_{e-s} = \dot{K}_u \cdot \frac{t}{E_o A_o} \qquad (3)$$

where $K_{e\text{-}s}$ is the "input voltage – output mechanical strain" coefficient, K_u is the "voltage" transmission coefficient, E_o is the Young's module for air, A_o is the area of acoustic beam in the air gap.

It is necessary to remember that both the transmission matrices and coefficients are frequency dependent, however, the corresponding notation is often omitted for convenience.

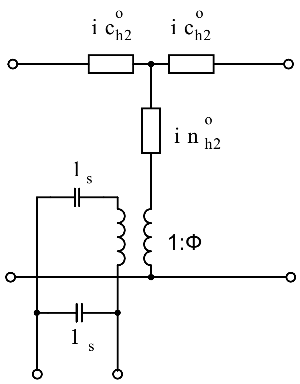
4. EQUIVALENT NETWORKS OF Q-SWITCH ELEMENTS

Each passive acoustic layer such as counterweight 1, glue layer 2, electrode 3, passive piezoceramics layer 9, top glass plate 10, glass prism 11, air gap 12, and bottom glass prism 13, may be described as a piece of transmission line with the transmission matrix being

$$\mathbf{A}_l = \begin{bmatrix} \cos(k_l d_l) & j \cdot Z_l \cdot \sin(k_l d_l) \\ \dfrac{j \cdot \sin(k_l d_l)}{Z_l} & \cos(k_l d_l) \end{bmatrix}, \qquad (4)$$

where d_l is the layer thickness, $k_l = 2\pi f / v_l$ is acoustical wave number, $Z_l = A_l \rho_l v_l$ is acoustic impedance, A_l is the area of acoustic beam in the layer, ρ_l and v_l are the layer's material density and acoustic velocity.

The equivalent networks for working transducer 6 (the transducer that actually converts the electric signal into the acoustic pulse) is a well-known Mason's network shown in Fig. 5 [Ref. 3].



$$C_0 = \frac{\varepsilon_{pe}\varepsilon_0 A_{ad}}{d_{pe}}$$

$$\Phi = k_{em}\sqrt{2 Z_{pe}^D C_0 f_{pe}}$$

$$Zt_{pe}^D = i \cdot Z_{pe}^D \tan\left(\frac{k_{pe}^D d_{pe}}{2}\right)$$

$$Z2_{pe}^D = \frac{-i \cdot Z_{pe}^D}{\sin\left(k_{pe}^D d_{pe}\right)}$$

Fig. 5
Mason's equivalent network for a piezoelectric transducer

K_m is the electromechanical coupling coefficient, ε_{pe} is the transducer's dielectric constant, f_{pe} is the transducer's central frequency,

Passive or active transducers (4 or 7) are transducers that convert acoustic waves into electrical signals. Their equivalent network is shown in Fig. 6.

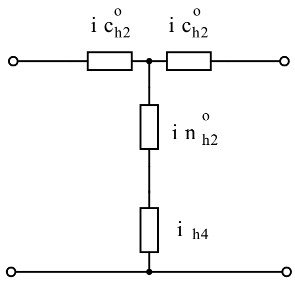
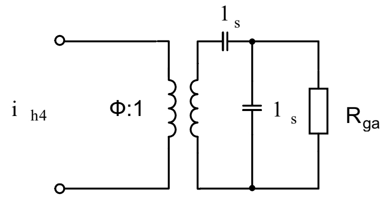


Fig. 6
Equivalent network for active or passive transducer. Z_{pt} is shown in Fig. 7

It has the following transmission matrix [Ref 5]:

$$\mathbf{A}_{4dr7} = \begin{bmatrix} t + \dfrac{Zt_{pe}^D}{Z2_{pe}^D + Z_{pt}} & 2Zt_{pe}^D + \dfrac{\left(Zt_{pe}^D\right)^2}{Z2_{pe}^D + Z_{pt}} \\ \dfrac{t}{Z2_{pe}^D + Z_{pt}} & t + \dfrac{Zt_{pe}^D}{Z2_{pe}^D + Z_{pt}} \end{bmatrix} \qquad (5)$$

The impedance Z_{pt} may be found from its equivalent network shown in Fig. 7 [Ref. 5].



$$\mathbf{A}_{pt} = \begin{bmatrix} \Phi & 0 \\ 0 & \dfrac{t}{\Phi} \end{bmatrix} \cdot \begin{bmatrix} t & \dfrac{j}{2\pi \cdot f \cdot C_0} \\ j \cdot 2\pi \cdot f \cdot C_0 & 0 \end{bmatrix}$$

Fig. 7
Equivalent network for the impedance Z_{pt} and its transmission matrix. R_{ga} is the output impedance of electric signal generator.

From Fig. 7 Z_{pt} may be found as the input impedance of the equivalent network [Ref. 6]:

$$Z_{pt} = \frac{\mathbf{A}_{pt_{tt}} \cdot R_{ga} + \mathbf{A}_{pt_{t2}}}{\mathbf{A}_{pt_{2t}} \cdot R_{ga} + \mathbf{A}_{pt_{22}}} \qquad (6)$$

Each active or passive transducer module also includes electrode 3 and two glue layers 2. Therefore, the overall transmission matrix for such module may be written as

$$\mathbf{A}_{5or8} = \mathbf{A}_g \cdot \mathbf{A}_e \cdot \mathbf{A}_g \cdot \mathbf{A}_{4or7} \qquad (7)$$

5. TRANSFORMED Q-SWITCH EQUIVALENT NETWORK AND ITS OVERALL TRANSMISSION MATRIX

As it has been mentioned earlier, the working transducer is a six-pole network. But for computation convenience, the Q-switch block diagram may be transformed is a such way that it will consist of four-pole elements only. As acoustic processes behind the working transducer are of no importance for the simulation, all the acoustic layers from the counterweight to working transducer may be substituted by their equivalent acoustic impedance. The transformed Q-switch equivalent network is shown in Fig. 8, where the acoustic impedance of Q-switch top part Z_{uh} may be found from Fig. 9 (Equation 16).

The transformed Q-switch equivalent network consists of the following elements:

1. Static capacitances. Their transmission matrix is the following [Ref. 5]:

$$\mathbf{A}_{C_0} = \begin{bmatrix} 1 & \dfrac{j}{2\pi \cdot f \cdot C_0} \\ j \cdot 2\pi \cdot f \cdot C_0 & 0 \end{bmatrix} \qquad (9)$$

2. Electromechanical transformer. Its transmission matrix is the following [Ref. 5]:

$$\mathbf{A}_{\mathrm{pt}} = \begin{bmatrix} \dfrac{1}{\Phi} & 0 \\ 0 & \Phi \end{bmatrix} \qquad (10)$$

3. Piezoelement's lower part. Its transmission matrix is the following [Ref. 5]:

$$\mathbf{A}_{pe} = \begin{bmatrix} 1 + \dfrac{Z1_{pe}^{D}}{Z_{uh}} & Z2_{pe}^{D} + Z1_{pe}^{D} + \dfrac{Z2_{pe}^{D} Z1_{pe}^{D}}{Z_{\mathrm{uh}}} \\ \dfrac{1}{Z_{uh}} & 1 + \dfrac{Z1_{pe}^{D}}{Z_{uh}} \end{bmatrix} \qquad (11)$$

4. All acoustic layers between the piezoelement and air gap [Ref. 6]:

$$\mathbf{A}_{14} = \ldots \cdot \mathbf{A}_{8} \ldots \cdot \mathbf{A}_{g} \cdot \mathbf{A}_{e} \cdot \mathbf{A}_{g} \cdot \mathbf{A}_{ll} \cdot \mathbf{A}_{p} \cdot \mathbf{A}_{c} \cdot \mathbf{A}_{o}$$

(12)The overall transmission matrix may be found as [Ref. 6]:

$$\mathbf{A}_{\Sigma} = \mathbf{A}_{C_0} \cdot \mathbf{A}_{\mathrm{pt}} \cdot \mathbf{A}_{pe} \cdot \mathbf{A}_{14} \qquad (13),$$

and the "voltage" transmission coefficient may be found as [Ref. 6]:

$$\dot{K}_{\mathrm{u}} = \frac{Z_1}{\mathbf{A}_{\Sigma_{11}} \cdot Z_1 + \mathbf{A}_{\Sigma_{12}} + \mathbf{A}_{\Sigma_{21}} \cdot Z_1 \cdot R_{\mathrm{g}} + \mathbf{A}_{\Sigma_{22}} \cdot R_{\mathrm{g}}} \qquad (14)$$

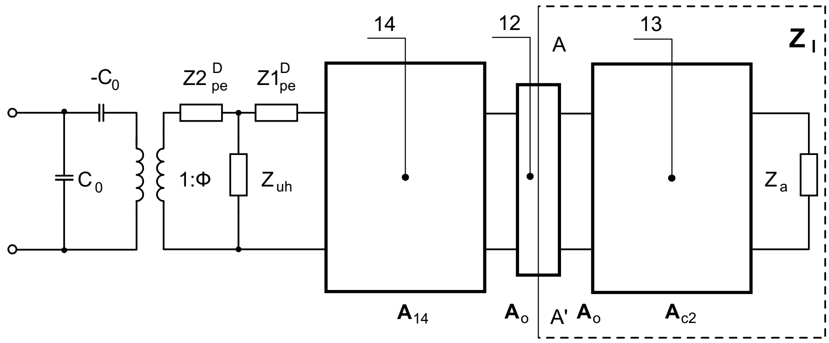
wherein the acoustic load impedance Z_l may be found as [Ref. 6]:

$$Z_1 = \frac{(\mathbf{A}_o \cdot \mathbf{A}_{c2})_{11} \cdot Z_{\mathrm{a}} + (\mathbf{A}_o \cdot \mathbf{A}_{c2})_{12}}{(\mathbf{A}_o \cdot \mathbf{A}_{c2})_{21} \cdot Z_{\mathrm{a}} + (\mathbf{A}_o \cdot \mathbf{A}_{c2})_{22}} \qquad (15),$$

and R_g is the output impedance of signal source.

Such "voltage" coefficient should be computed for each transducer and for each acoustic mode. The transmission coefficients $K_{e\text{-}s}$ may be obtained by substituting K_u into Equation 3,.

Now, the output signal of each partial system may be obtained as the inverse Fourier transform of the product between the input signal spectrum and transmission coefficient [Ref. 7].

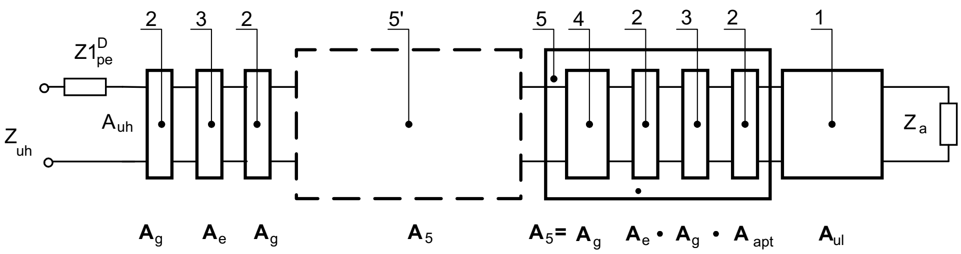


14. Acoustic layers between the working transducer and air gap
13. Bottom acoustic layers

Z_{uh} is the acoustic impedance of Q-switch top part (Fig. 9)

$\boldsymbol{Z_l}$ is the acoustic load

Fig. 8
Transformed Q-switch equivalent network



$$\mathbf{A}_{uh} = \mathbf{A}_{g} \cdot \mathbf{A}_{e} \cdot \mathbf{A}_{g} \cdot \ldots \cdot \mathbf{A}_{8} \cdot \ldots \cdot \mathbf{A}_{ul}$$

$$Z_{\mathrm{uh}} = \frac{\mathbf{A}_{uh_{11}} \cdot Z_{\mathrm{a}} + \mathbf{A}_{uh_{12}}}{\mathbf{A}_{uh_{21}} \cdot Z_{\mathrm{a}} + \mathbf{A}_{uh_{22}}} + Z1_{pe}^{D} \qquad (16)$$

Fig. 9
Q-switch top part

6. Q-SWITCH INPUT IMPEDANCE. CALCULATION AND MEASUREMENTS.

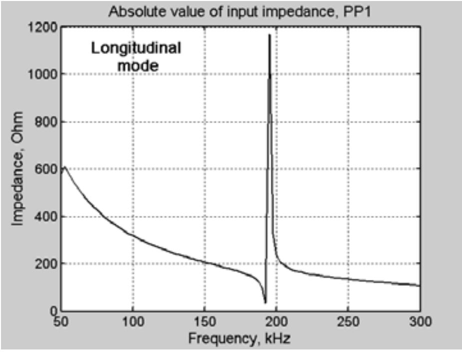
The best way to verify the simulation model of an electrical network is to compare its calculated and measured input impedance.

The input impedance of the Q-switch equivalent network may be found as [Ref. 6]:

$$Zin_{\mathrm{pe}} = \frac{\mathbf{A}_{\Sigma 1_{11}} \cdot Z_1 + \mathbf{A}_{\Sigma 1_{12}}}{\mathbf{A}_{\Sigma 1_{21}} \cdot Z_1 + \mathbf{A}_{\Sigma 1_{22}}} \qquad (17)$$

where $\mathbf{A}_{\Sigma 1} = \mathbf{A}_{C_0} \cdot \mathbf{A}_{\mathrm{pt}} \cdot \mathbf{A}_{pe} \cdot \mathbf{A}_{14}$.

The calculated input impedance for the first transducer is shown in Fig. 10. The input impedance was calculated separately for the longitudinal and slender acoustic modes. Its measured impedance is shown in Fig. 11. Both calculations and measurements are in good agreement. Measurements and calculations for other transducers also have shown that experimental and theoretical results coincide quite well. That means that the simulation model is correct.



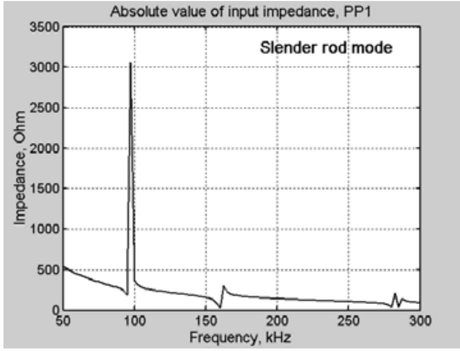
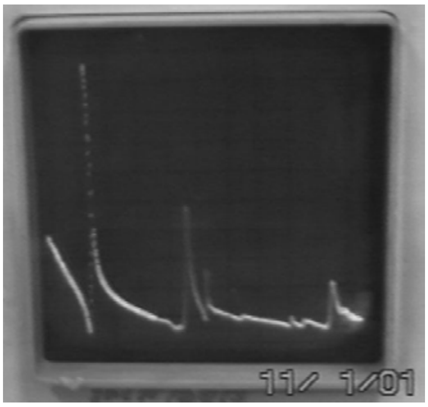


Fig. 10
Q-Switch input impedance, (first transducer, calculated)



Peaks (from left to right)

1. 85 kHz – slender rod mode
2. 168 kHz – longitudinal mode
3. 185 kHz - glue and electrodes
4. 280 kHz - slender rod mode

Fig. 11
Q-Switch input impedance, (first transducer, measured)

7. Q-SWITCH INPUT ELECTRICAL SIGNAL AND ITS SPECTRUM

The input electrical signal shown in Fig. 2 may be represented as the sum of three partial signals:

$$s(t) = s1(t) + s2(t) + s3(t) \qquad (17)$$

wherein:

1. $s1(t) = U \cdot (1 - \exp(-\alpha 1 \cdot t))$ within $[0, t_1]$ is the pulse front edge, $\alpha 1 = \frac{2e}{t_r}$; $t_1 = t_r$, t_r is the pulse rise time;
2. $s2(t) = U$ within $(t_1, t_2]$ is the pulse steady component, $t_2 = t_r + t_s$, t_s is the pulse steady time,
3. $s3(t) = U \cdot (\exp(-\alpha 3(t - t_2)))$ within (t_2, ∞) is the pulse decay, $\alpha 3 = \frac{2e}{t_d}$, t_d is the pulse decay time

The resulting signal is shown in Fig. 12.

The spectrum of such signal may be found as the sum of the spectrums of the partial signals [Ref. 7]:

$$S(\omega) = S1(\omega) + S2(\omega) + S3(\omega) \qquad (18)$$

wherein:

1. $S1(\omega)$ is the pulse rise spectrum:

$$S1(\omega) = \left[U \cdot t_1 \cdot \mathrm{sinc}\left(\frac{\omega t_1}{2}\right) \cdot \exp\left(-j\frac{\omega t_1}{2}\right)\right] + \\ + \frac{U}{\alpha 1 + j\omega} \cdot \left[\exp(-t_1(\alpha 1 + j\omega)) - 1\right]$$

2. $S2(\omega)$ is the pulse steady component spectrum:

$$S2(\omega) = U \cdot (t_2 - t_1) \cdot \mathrm{sinc}\left(\frac{\omega \cdot (t_2 - t_1)}{2}\right) \cdot \exp\left(-j\frac{\omega(t_2 - t_1)}{2}\right)$$

3. $S2(\omega)$ is the pulse decay spectrum:

$$S3(\omega) = \frac{U}{\alpha 1 + j\omega} \cdot \exp(-j\omega t_2),$$

and $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$, $\omega = 2\pi f$.

The resulting spectrum is shown in Fig. 13.

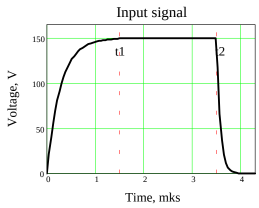


Fig. 12.
Q-Switch input signal

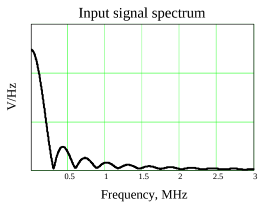


Fig. 13.
Q-Switch input spectrum

8. Q-SWITCH OUTPUT SIGNAL. CALCULATION AND MEASUREMENTS.

The overall output acoustical signal may be found as a sum of partial output signals from each 6 partial systems computed for each of two modes. Using the well known Fourier transform properties, the gap strain may be found as:

$$s_o(t) = \frac{1}{E_o \cdot A_o} \cdot F^{-1}\left[S(\omega) \cdot \sum_N \left(\dot{K}_u^L(N,f) + \dot{K}_u^S(N,f)\right)\right] \qquad (19)$$

where $F^{-1}()$ is the inverse Fourier transform, $\dot{K}_u^L(N,f)$ and $\dot{K}_u^S(N,f)$ are the "voltage" transmission coefficients for the longitudinal (L) and slender rod mode (S) respectively, N is the working transducer number. The computed gap strain (Eq. 19) and thickness (Eq. 2) are shown in Fig. 14 and 15, respectively. It is worth noticing that although $s_o(t)$ may be less than 0, the gap thickness cannot be negative, therefore, the minimal thickness is restricted to $0.01 \cdot d_o$.

The computed (Eq. 1) and measured output optical signal is shown in Fig. 16 and 17, respectively. They are in a good agreement. More sharp attenuation of the computed signal may be explained by energy leak of the slender rod mode into the signal source, because its output impedance is not large enough.

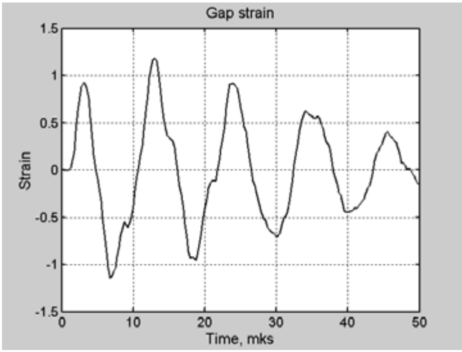


Fig. 14
Q-switch gap strain

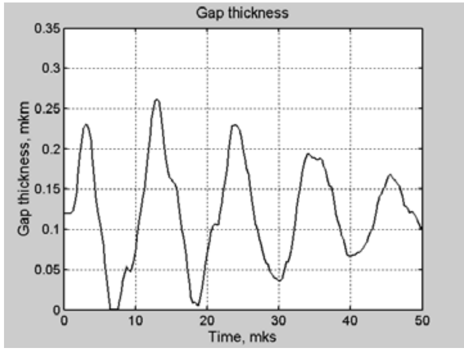


Fig. 15
Q-switch gap thickness

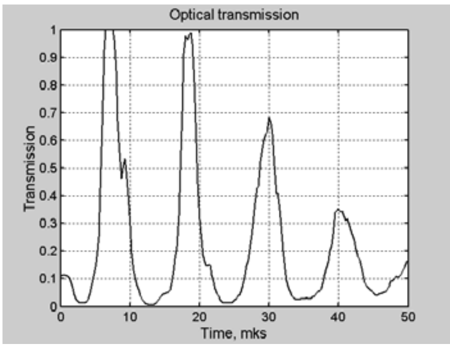


Fig. 16
Q-switch output optical signal (computed)

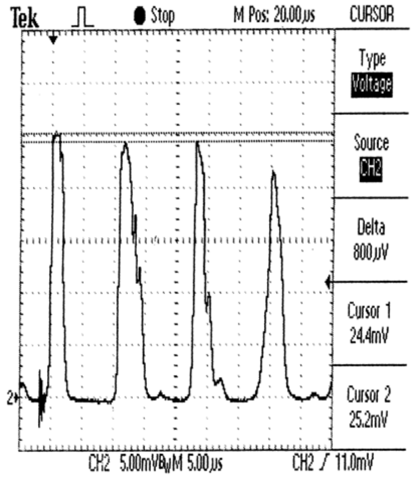


Fig. 17
Q-switch output optical signal (measured)

8. CONCLUSIONS

The presented results show that the method of equivalent four-pole networks can be used to simulate rather complex acoustic systems. The computational results are in a good agreement with experimental measurements.

Also, the advantage of this method is that it operates with data types, 2x2 matrices, that are very easy for contemporary computers to process. This considerably increases computational speed.

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